

Spin effects in the electron structure functions[★]

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Abstract. The electron structure functions are studied in polarized e^+e^- scattering. The formulae for longitudinally and transversely polarized electrons are presented. The smallness of the electron mass leads to negligible cross-sections and asymmetries in some cases. Positivity constraints on the structure functions and parton densities are constructed and discussed. The cross-section asymmetries at very high energies, where the inclusion of all electroweak bosons is necessary, are calculated. Numerical examples, using the asymptotic solutions for the parton densities inside the electron, are presented.

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1 Introduction

The electron structure function has been constructed and studied in a series of papers [1–6], and it has been first measured at LEP [7]. It became a useful notion at present energies where it can serve, among others, as a cross-check of the extensively studied photon structure function [8,9]. At very high energies its use is without doubt necessary: it includes non-negligible contributions from all intermediate bosons and their $(\gamma-Z)$ interference. A detailed comparison with the “photon structure approach” has been presented in [6]. In this paper we study the spin dependence of the partonic content of the electron/positron in polarized deep-inelastic $e^+e^- \rightarrow eX$ scattering. The structure of the process can be visualized as in Fig. 1. The electron e_λ of polarization $\lambda = \pm$ and momentum $l = (E, \mathbf{l})$ probes the positron e_κ of polarization $\kappa = \pm$ and momentum k by emitting the photon or Z boson (the reverse setup where the positron probes the electron is analogical but differs quantitatively when weak interactions are included). The scattered electron is tagged so that we can determine the relevant variables by measuring its final momentum $l' = (E_{\text{tag}}, \mathbf{l}')$ and scattering angle θ_{tag} . The target positron remains untagged – its momentum transfer $P^2 = -(k - k')^2$ is smaller than the experimentally determined upper limit P_{max}^2 . The hadronic structure developed by the target lepton begins with the

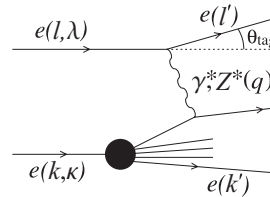


Fig. 1. Electron–positron scattering and the electron structure

emission of a colinear electroweak boson: photon, Z or W (the anti-tag condition cannot be imposed when the W boson is produced).

In Sect. 2 we limit ourselves to the energy range where both the probe and target leptons emit photons only. We introduce there the cross-sections and asymmetries in analogy to electron–nucleon scattering. The positivity constraints among structure functions and parton densities inside the electron are presented in Sect. 3. In Sect. 4 we generalize the discussion to the case where all electroweak bosons are taken into account. We include the γ , Z and $\gamma-Z$ emission by the probe lepton and the contribution of γ , Z , $\gamma-Z$ and W to the lepton structure function. We illustrate the calculation with numerical examples where we take the parton densities inside the electron from our asymptotic solutions of the evolution equations [1–6]. Section 5 summarizes our results.

2 Cross-sections and asymmetries

Defining the deep-inelastic scattering variables in the standard way,

$$Q^2 = -(l - l')^2 = 2 E E_{\text{tag}} (1 - \cos \theta_{\text{tag}}), \quad (1)$$

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$$z = \frac{Q^2}{2kq} = \frac{\sin^2(\theta_{\text{tag}}/2)}{E/E_{\text{tag}} - \cos^2(\theta_{\text{tag}}/2)},$$

the cross-section for unpolarized electron–positron scattering can be written in the limit $Q^2 \gg P_{\text{max}}^2$

$$\begin{aligned} \frac{d^2\sigma_{ee}}{dzdQ^2} &\equiv \frac{1}{2} \frac{d^2\sigma_{e_+e_+} + d^2\sigma_{e_-e_+}}{dzdQ^2} \\ &= \frac{2\pi\alpha^2}{zQ^4} \left[(1 + (1-y)^2) F_2^e(z, Q^2, P_{\text{max}}^2) \right. \\ &\quad \left. - y^2 F_L^e(z, Q^2, P_{\text{max}}^2) \right], \end{aligned} \quad (2)$$

with

$$y = \frac{qk}{lk} = 1 - (E_{\text{tag}}/E) \cos^2(\theta_{\text{tag}}/2). \quad (3)$$

Taking into account the smallness of the electron mass m , the longitudinal structure function reads

$$\begin{aligned} F_L(z, Q^2, P_{\text{max}}^2) &= \frac{1}{2z} \left(\frac{4m^2 z^2}{Q^2} + 1 \right) F_2(z, Q^2, P_{\text{max}}^2) - F_1(z, Q^2, P_{\text{max}}^2) \\ &\simeq \frac{1}{2z} F_2(z, Q^2, P_{\text{max}}^2) - F_1(z, Q^2, P_{\text{max}}^2) \\ &\equiv R F_1(z, Q^2, P_{\text{max}}^2), \end{aligned} \quad (4)$$

and the cross-section (2) reduces to

$$\begin{aligned} \frac{d^2\sigma_{ee}}{dzdQ^2} &= \frac{4\pi\alpha^2}{Q^4} \left[\left(1 + (1-y)^2 \right) (1+R) - \frac{1}{2z} y^2 R \right] \\ &\quad \times F_1^e(z, Q^2, P_{\text{max}}^2). \end{aligned} \quad (5)$$

The structure function F_1 is related to the quark densities q_i inside the electron in the standard way:

$$F_1(z, Q^2, P_{\text{max}}^2) = \sum_i e_{q_i}^2 q_i(z, Q^2, P_{\text{max}}^2), \quad (6)$$

where e_{q_i} is the i th quark fractional charge. The polarized cross-section reads

$$\begin{aligned} \frac{d^2\Delta\sigma_{ee}}{dzdQ^2} &\equiv \frac{1}{2} \frac{d^2\sigma_{e_+e_+} - d^2\sigma_{e_-e_+}}{dzdQ^2} \\ &= \frac{-4\pi\alpha^2}{Q^4} \left[(1 - (1-y)^2) g_1(z, Q^2, P_{\text{max}}^2) \right. \\ &\quad \left. - 2zy\epsilon g_2(z, Q^2, P_{\text{max}}^2) \right] \\ &\simeq \frac{-4\pi\alpha^2}{Q^4} (1 - (1-y)^2) g_1(z, Q^2, P_{\text{max}}^2), \end{aligned} \quad (7)$$

where we introduced

$$\epsilon \equiv \frac{2m^2}{s} \stackrel{\text{LAB}}{=} \frac{m}{E_{\text{LAB}}} \stackrel{\text{CM}}{=} \frac{m^2}{2E_{\text{CM}}} \simeq 0 \quad (8)$$

to demonstrate the terms present in electron–nucleon scattering and negligible in the present case due to the smallness of the electron mass.

Polarizing the spin \mathbf{S} of the probing electron in the transverse direction one arrives at a negligible cross-section ($\sim \sqrt{\epsilon}$), again due to the smallness of the electron mass (the angle ϕ is the angle between the $(\mathbf{k}, \mathbf{k}')$ and (\mathbf{k}, \mathbf{S}) planes):

$$\begin{aligned} \frac{d^2\Delta_{\perp}\sigma_{ee}}{dzdQ^2} &\equiv \frac{1}{2} \frac{d^2\sigma_{e_{\uparrow}e_+} - d^2\sigma_{e_{\downarrow}e_+}}{dzdQ^2} \\ &= \frac{-4\pi\alpha^2}{Q^4} \sqrt{2zy(1-y)} \sqrt{\epsilon} \\ &\quad \times \left(y g_1(z, Q^2, P_{\text{max}}^2) + 2g_2(z, Q^2, P_{\text{max}}^2) \right) \cos\phi. \end{aligned} \quad (9)$$

Taking into account the above results, the discussion of the relevant asymmetries simplifies. The spin–spin asymmetries used in deep-inelastic scattering,

$$A_{\parallel} = \left(\frac{d^2\Delta\sigma_{ee}}{dzdQ^2} \right) / \left(\frac{d^2\sigma_{ee}}{dzdQ^2} \right)$$

and

$$A_{\perp} = \left(\frac{d^2\Delta_{\perp}\sigma_{ee}}{dzdQ^2} \right) / \left(\frac{d^2\sigma_{ee}}{dzdQ^2} \right), \quad (10)$$

can be expressed in terms of virtual photon scattering asymmetries $A_{1,2}$ (to compare with the nucleon case see e.g. [10]):

$$A_{\parallel} = D(A_1 + \eta A_2) \quad \text{and} \quad A_{\perp} = d(A_2 + \xi A_1), \quad (11)$$

where

$$\begin{aligned} A_1 &= \frac{g_1 - \frac{2z}{y} \epsilon g_2}{F_1} \simeq \frac{g_1}{F_1}, \quad A_2 = \sqrt{\frac{2z}{y}} \sqrt{\epsilon} \frac{g_1 + g_2}{F_1} \simeq 0, \\ D &= \frac{1 - \kappa(1-y)}{1 + \kappa R} 2(1-y) + R[1 + (1-y)^2], \\ d &= D \sqrt{\frac{2\kappa}{1-\kappa}}, \\ \kappa &= \frac{1 + (1-y)^2}{2(1-y)}, \quad \xi = \eta \frac{1 + \kappa}{2\kappa} \simeq 0, \end{aligned} \quad (12)$$

$$\eta = \frac{\sqrt{2zy} [1 + (1-y)^2]}{(1-y) [1 - (1-y)^2]} \sqrt{\epsilon} \simeq 0. \quad (13)$$

From the above definitions one sees that the structure function g_1 is directly measured by the asymmetry A_{\parallel} whereas the structure function g_2 is practically inaccessible in e^+e^- scattering.

3 Positivity constraints

The structure functions $F_{1,2}$ and $g_{1,2}$ are related to the forward Compton scattering amplitudes $M_{a\lambda b\mu}$ and photoabsorption cross-sections $\sigma_{T,S,ST}^{(\pm)}$. In our notation a (b)

and λ (μ) are the helicities of the initial (final) photon and positron:

$$\begin{aligned}\frac{1}{m}F_1 &= \frac{1}{2}(M_{1+1+} + M_{1-1-}) \sim \sigma_T^+, \\ \frac{1}{m}F_L &= M_{0+0+} \sim \sigma_S, \\ \frac{1}{m}g_1 - \frac{2z}{y}\epsilon g_2 &= \frac{1}{2}(M_{1+1+} - M_{1-1-}) \sim \sigma_T^-, \\ \frac{2\sqrt{z}}{my}\sqrt{\epsilon}(g_1 + g_2) &= M_{1-0+} \sim \sigma_{TS}.\end{aligned}\quad (14)$$

The structure of the above relations is analogical to that of the deep-inelastic electron–nucleon scattering due to the identical spin structure of both processes. Therefore we can easily derive the constraints on the structure functions coming from the positivity of the photoabsorption cross-sections [11,12]. The first group can be written as

$$\begin{aligned}F_1 &\geq 0, \\ F_L &= \frac{1}{2z}\left(\frac{4m^2z^2}{Q^2} + 1\right)F_2 - F_1 \geq 0, \\ \sqrt{\frac{R(1+A_1)}{2}} &\geq A_2.\end{aligned}\quad (15)$$

The second type of relation introduces constraints among the parton densities inside the electron and the chirality-odd, twist 2 structure function h_1 [13]. Derived first by Soffer [14] for the nucleon, the relation takes an analogical form in the electron case:

$$q(z) + \Delta q(z) \geq 2 |h_1(z)|. \quad (16)$$

The inequality (16) holds for each quark (antiquark) flavor separately. It seems to be fulfilled trivially since h_1 , representing the parton transversity distributions, does not evolve from the initial electrons through the photons.

4 Cross-section and asymmetries with all electroweak bosons

In the presence of all electroweak bosons the picture of e^+e^- scattering becomes more complicated than in the pure QED approach. The structure functions F_1 and g_1 depend in general on both the polarization of the probe and the target. In addition, the deep-inelastic scattering depends now on which lepton is the target and which is the probe. But as in the previous sections, due to the smallness of the electron mass, many terms can be omitted without loss of accuracy. Out of all possible polarization directions only the one along the momentum of the leptons is non-negligible.

The cross-section for the scattering of electron of helicity $\lambda = \pm$ off the positron of helicity $\kappa = \pm$ reads

$$\frac{d\sigma(e_\lambda e_\kappa)}{dz dQ^2} = 4\pi\alpha^2 \sum_{B,B'} \frac{\hat{g}_{Be_\lambda}}{Q^2 + M_B^2} \frac{\hat{g}_{B'e_\lambda}}{Q^2 + M_{B'}^2}$$

$$\times [Y_+(y)J_{BB'}^{\lambda\kappa} + Y_-(y)J_{BB'}^{-\lambda\kappa}], \quad (17)$$

where $M_B, M_{B'}$ are the masses of the exchanged bosons (γ or Z), \hat{g}_{Be_λ} the lepton–boson electroweak couplings,

$$Y_-(y) = 1, \quad Y_+(y) = (1-y)^2 \quad (18)$$

and

$$\begin{aligned}J_{BB'}^{\lambda\kappa} &= \hat{g}_{Bq-\lambda}\hat{g}_{B'q-\lambda}f_{q-\lambda}^{e_\kappa}(z, Q^2, P_{\max}^2) \\ &\quad + \hat{g}_{Bq\lambda}\hat{g}_{B'q\lambda}f_{\bar{q}-\lambda}^{e_\kappa}(z, Q^2, P_{\max}^2).\end{aligned}\quad (19)$$

In the latter formula the $f_{q_\lambda}^{e_\kappa}(z, Q^2, P^2)$ represent the densities of quarks of helicity λ inside the positron of helicity κ .

By defining

$$Y = Y_+ + Y_-, \quad \Delta Y = Y_+ - Y_- \quad (20)$$

and

$$\begin{aligned}F_1^{\lambda\kappa} &= \sum_{B,B'} \hat{g}_{Be_\lambda}\hat{g}_{B'e_\lambda} \frac{Q^2}{Q^2 + M_B^2} \frac{Q^2}{Q^2 + M_{B'}^2} [J_{BB'}^{+\kappa} + J_{BB'}^{-\kappa}],\end{aligned}\quad (21)$$

$$\begin{aligned}g_1^{\lambda\kappa} &= \lambda \sum_{B,B'} \hat{g}_{Be_\lambda}\hat{g}_{B'e_\lambda} \frac{Q^2}{Q^2 + M_B^2} \frac{Q^2}{Q^2 + M_{B'}^2} [J_{BB'}^{+\kappa} - J_{BB'}^{-\kappa}],\end{aligned}\quad (22)$$

we can write the cross-section (17) as follows:

$$\frac{d\sigma(e_\lambda e_\kappa)}{dz dQ^2} \equiv \frac{2\pi\alpha^2}{Q^4} [Y(y)F_1^{\lambda\kappa} + \Delta Y(y)g_1^{\lambda\kappa}]. \quad (23)$$

In the pure QED case, where the only boson contributing to the process is the photon (both the highly virtual one from the beam electron and the nearly real one from the target positron), the cross-section reduces to the formulae (2) and (7).

To see what effects introduce the inclusion of all electroweak bosons we calculated the above cross-sections numerically using as the parton densities the asymptotic solutions of our evolution equations [2–6]. In Fig. 2 the e^-e^+ scattering cross-section is presented at $\sqrt{s} = 1$ TeV, $Q^2 = 5 \times 10^5$ GeV² and $P_{\max}^2 = 2 \times 10^3$ GeV² for various helicity configurations. Large differences result from different electroweak couplings of γ, W and Z bosons. In Fig. 3 we present the largest asymmetry which can be constructed from the above cross-sections:

$$\begin{aligned}A &= \left(\frac{d\sigma(e_+e_-) - d\sigma(e_-e_+)}{dz dQ^2} \right) \\ &\quad / \left(\frac{d\sigma(e_+e_-) + d\sigma(e_-e_+)}{dz dQ^2} \right).\end{aligned}\quad (24)$$

For comparison we present the same asymmetry with the photon only. The presence of weak bosons is clearly seen.

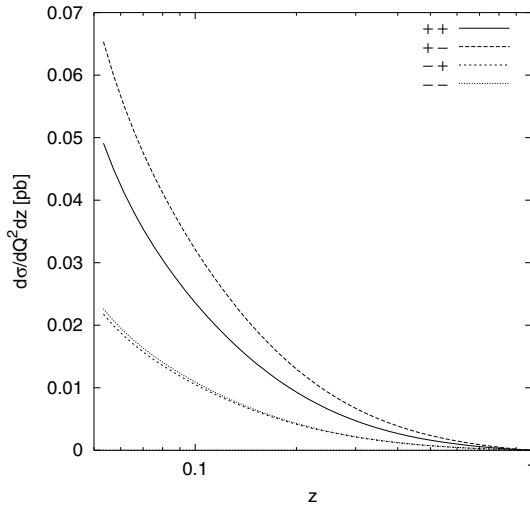


Fig. 2. The cross-section $\frac{d\sigma(e_\lambda e_\kappa)}{dz dQ^2}$ as a function of z for $\sqrt{s} = 1$ TeV, $Q^2 = 5 \times 10^5$ GeV², $P_{\max}^2 = 2 \times 10^3$ GeV² with the (electron, positron) helicity (λ, κ)

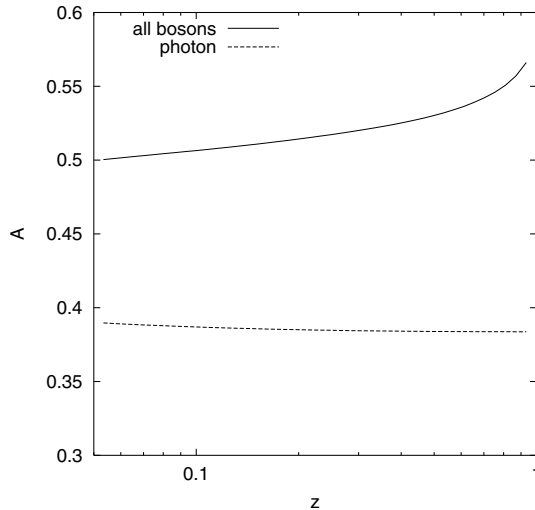


Fig. 3. The asymmetry (24) as a function of z with all electroweak bosons included in the electron structure function (solid line) and the photon only (dashed line)

5 Summary

We have completed the analysis of the QCD structure of leptons by looking at the spin effects which appear in e^+e^- collisions at present and TeV energies. As compared to electron–nucleon scattering, where the spin structure of the process is analogical, many formulae simplify due to the smallness of the electron mass. On the other hand new effects can be observed at very high energies, in particular all electroweak gauge bosons play an important role in building up the QCD structure. The use of the electron/positron structure functions which include also interference effects is then necessary. We also give numerical estimates of the size of these new effects at TeV energies. Since no parton parameterizations exist at these energies, we use our asymptotic solutions of the corresponding evolution equa-

tions. The contribution from weak gauge bosons turns out to be quantitatively significant and should be visible in the next generation of e^+e^- experiments.

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